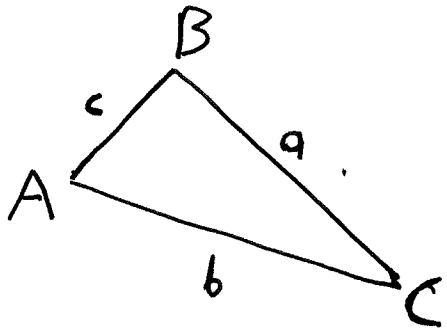
Sine Law

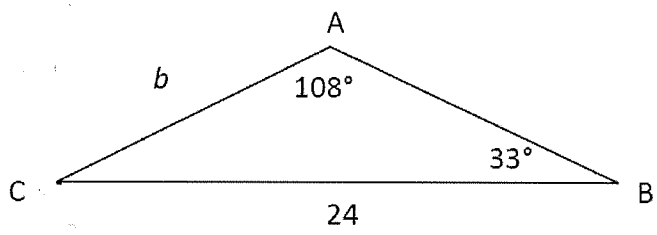
Sine law can be used to solve for a side or an angle in non-right triangles. We use it when we have either two angles and one side OR two sides and a non-enclosed angle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Use Sine Law to determine the length of side b:

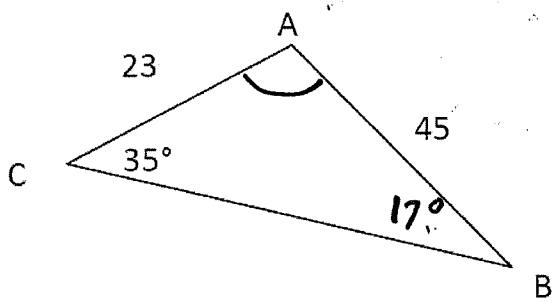


$$\frac{b}{\sin 33^\circ} = \frac{24}{\sin 108^\circ}$$

$$b = \frac{24}{\sin 108^\circ} \times \sin 33^\circ = 13.74$$

$$\approx 14$$

Use Sine Law to determine angle A



$$\frac{\sin B}{23} = \frac{\sin 35^\circ}{45}$$

$$\sin B = \frac{\sin 35^\circ}{45} \times 23$$

$$\sin B = 0.29$$

$$B = 17^\circ$$

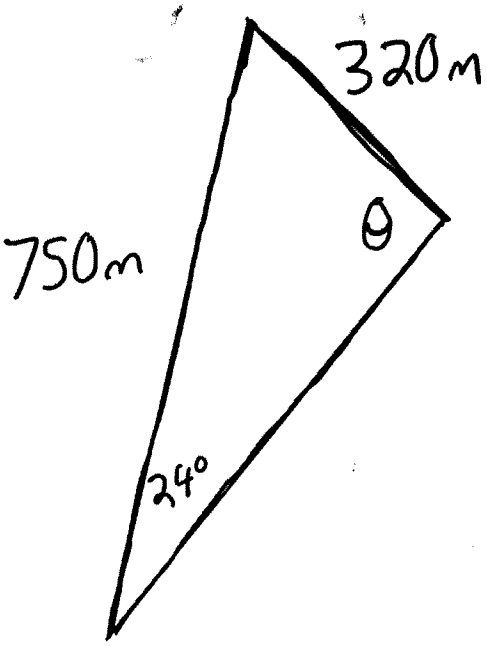
$$180 - 35^\circ - 17^\circ$$

$$= 128^\circ$$

Ambiguous case warning

Using sine law to determine an angle will always return an angle between 0 and 90 degrees. If the angle you are looking for is between 90 and 180 degrees, you need to calculated the angle in second quadrant with the reference angle you found by taking 180- θ_{ref}.

Example



$$\frac{\sin\theta}{750m} = \frac{\sin 24^\circ}{320m}$$

$$\sin\theta = \frac{\sin 24^\circ}{320} \times 750$$

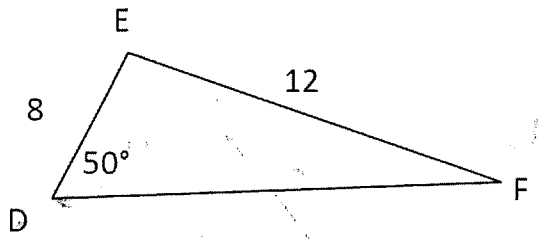
$$\theta = 72^\circ$$

But θ is > 90

so $\theta = 180^\circ - 72^\circ$

$= 108^\circ$

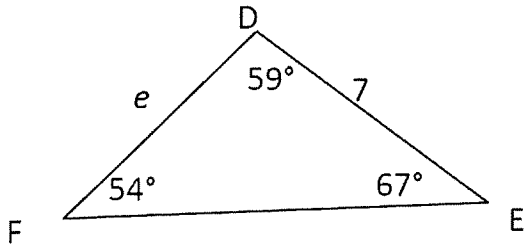
Practice



Determine the measure of angle F

$$\frac{\sin F}{8} = \frac{\sin 50}{12}$$

$$F = 30.71^\circ \approx 30^\circ$$



Determine the length of side e

$$\frac{e}{\sin 67} = \frac{7}{\sin 54}$$

$$e = 7.96$$

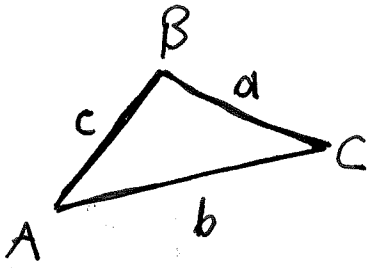
$$\approx 8$$

Cosine Law

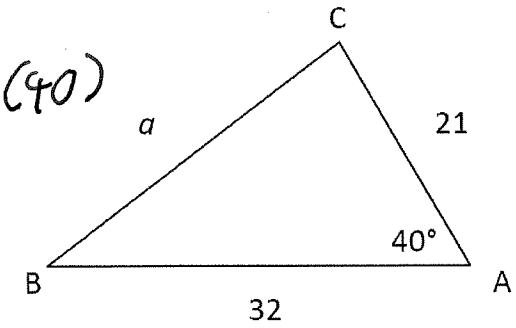
Like sine law, the cosine law can be used to solve for a side or an angle in non-right triangles. We use it when we have three sides OR two sides and the included angle.

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$



Use Cosine Law to determine the length of side a

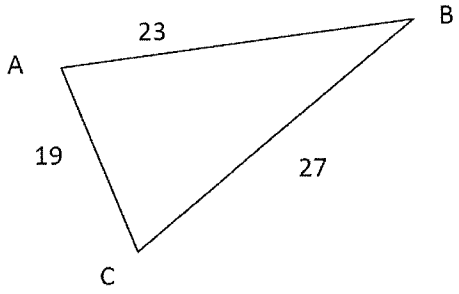


$$a^2 = 21^2 + 32^2 - 2(21)(32)\cos(40)$$

$$= 435$$

$$a = 20.87$$

$$\approx 20$$



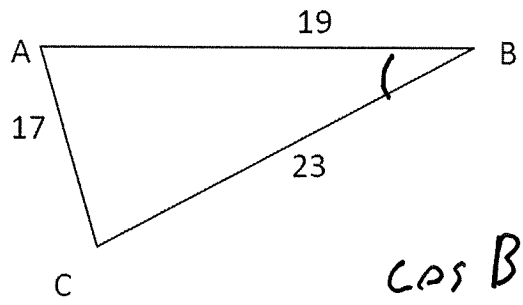
Use Cosine Law to determine angle A

$$\cos A = \frac{27^2 - 23^2 - 19^2}{-2(23)(19)}$$

$$= 0.1842$$

$$A = 79°$$

Practice

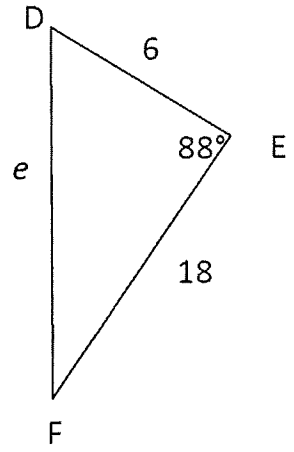


Determine the measure of angle B.

$$\cos B = \frac{17^2 - 19^2 - 23^2}{-2(19)(23)}$$

$$B = 47^\circ$$

Determine the length of side e



$$e^2 = 6^2 + 18^2 - 2(6)(18)\cos 88$$

$$e = 18.77$$

$$\simeq 19$$

Momentum

An object in motion will tend to stay in motion. What two factors affect how difficult it is to stop an object that is moving?

The momentum of an object is:

A bullet shot from a gun has a lot of momentum because

A freight train slowly moving in the train yard has a lot of momentum because

Example 1: Calculate the momentum of a 1100 kg car travelling along the highway at 15 m/s.

Example 2: Calculate the momentum of a 1.2 kg ball thrown at 13.9 m/s.

Example 3: Determine the change in momentum of a 48 kg runner who speeds up from 5.0 m/s to 7.0 m/s.